Symmetry crossover in quantum wires with spin-orbit interaction

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We systematically calculate localization length and conductance fluctuations in quantum wires with spin-orbit interaction to demonstrate that the effective symmetry of the system is determined by the relative magnitude between the spin-relaxation length and the localization length. When the localization length is much smaller than the spin-relaxation length, the localization length is close to the value of wires without spin-orbit interaction. When the localization length exceeds the spin-relaxation length, the localization length is enhanced and approaches that of wires with strong spin-orbit interaction. The same symmetry crossover occurs in conductance fluctuations.

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I. INTRODUCTION

Disordered systems are classified into three universality classes by the presence or absence of time-reversal and spin-rotational symmetry which are not destroyed by potential randomness.[1] The symmetry plays a significant role in determining localization effects and conductance fluctuations, directly observable in transport experiments.[2] In the presence of spin-orbit interaction, the spin-rotational symmetry is broken and the universality class changes from orthogonal to symplectic.[3] The purpose of this paper is to demonstrate the role of spin-relaxation length in determining this symmetry crossover in quantum wires with spin-orbit interaction.

In quantum wires, drastic enhancement of the localization length and reduction in conductance fluctuations by strong spin-orbit interaction have already been demonstrated both analytically[4,5] and numerically.[6,7] The important question remains, however, how and when the symmetry crossover occurs as a function of the strength of the spin-orbit interaction, the degree of disorder, the wire width, etc.

In a semiconductor two-dimensional (2D) system such as a quantum well and a heterostructure, the so-called structure inversion asymmetry causes spin splitting in the presence of spin-orbit interaction.[8–12] The term responsible for the spin splitting behaves as an effective magnetic field and causes spin precession, and generally leads to spin relaxation in presence of spin-independent impurity scattering.[13] In quasi-one-dimensional systems such as quantum wires, the spin relaxation is suppressed as has been suggested both theoretically[14–16] and experimentally,[17,18] which makes a quantum wire suitable for spintronics.[19,20] In a previous paper, a detailed numerical study was devoted to the spin-relaxation length in quantum wires.[21]

In this paper, we perform systematic calculations of the localization length and the conductance fluctuations in quantum wires, to demonstrate that the effective symmetry of the system is determined by the relative magnitude between the spin-relaxation length and the localization length. In Sec. II, the formulation of the problem is briefly given. In Sec. III, the numerical results are presented. It is shown that the localization length is given by that of wires without spin-orbit interaction when it is smaller than the spin-relaxation length, and that it is enhanced and becomes that of wires with strong spin-orbit interaction when it is larger than the spin-relaxation length. The same symmetry crossover occurs also in conductance fluctuations. A short summary is given in Sec. IV.

II. FORMULATION

The system to be considered is exactly the same as that used in a previous study[21] and therefore we shall briefly summarize the characteristic parameters. In semiconductor 2D systems, the presence of the so-called structure inversion asymmetry gives Hamiltonian

$$\mathcal{H}=\frac{\hbar^2\boldsymbol{k}^2}{2m^*}+\alpha(k_x\sigma_y-k_y\sigma_x), \tag{1}$$

where m^* is effective mass, $\boldsymbol{k}=(k_x,k_y)$ is wave vector, and σ_μ ($\mu=x,y,z$) are the Pauli spin matrices. The strength of spin splitting is characterized by δ which is defined as the ratio of the spin splitting at the Fermi level to the Fermi energy, i.e., $\delta=\alpha k_F/E_F$. In typical quantum wells, we have $\delta\lesssim 0.05$.

A quantum wire is constructed by confining electrons within a narrow strip with width W. We choose the x and y axis in the longitudinal and transverse direction of the wire, respectively. The channel number per spin of the wire, i.e., the number of occupied subbands below the Fermi level, is $N=[2W/\lambda_F]$, where $[x]$ is an integer part of x and λ_F is the Fermi wavelength of the 2D system with E_F.

In actual calculations, a square-lattice tight-binding model with lattice constant a is used to simulate quantum wires.[22] The effect of disorder is included by on-site potential which is uniformly distributed with width U. In continuum limit, the disorder potential can be understood as high concentration of δ-function potentials and U is characterized by mean-free path Λ in a 2D system as described in the previous paper.[21]

We consider a quantum wire with length L. At both ends of the wire, a spin-independent ideal lead with same width is attached. Using the recursive Green's-function technique, we

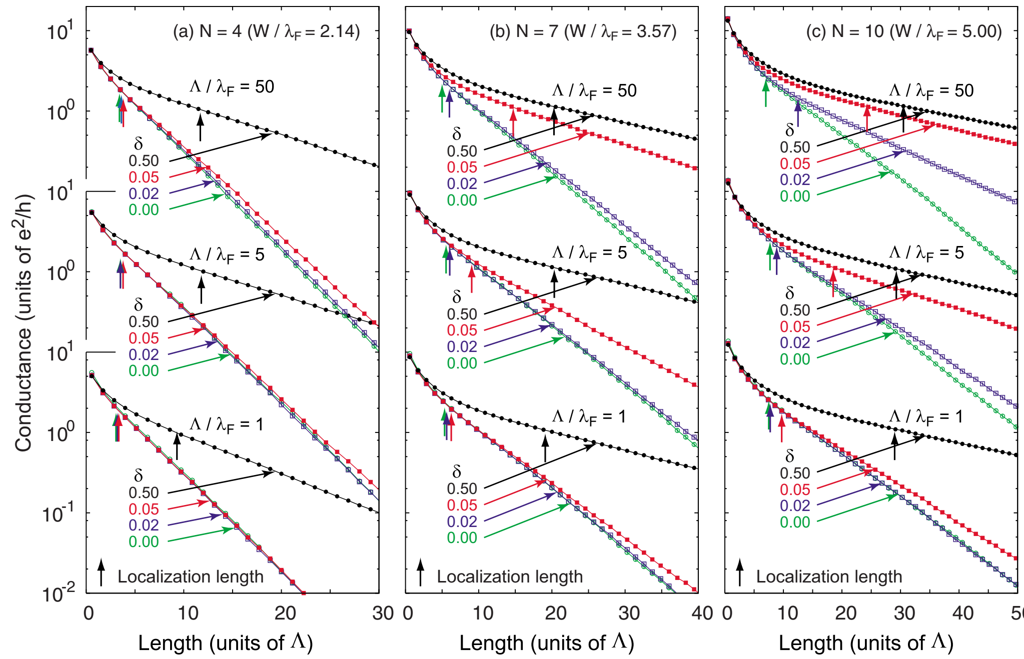

FIG. 1. (Color online) Calculated geometric average of the conductance as a function of length measured in units of mean-free path Λ for $\Lambda/\lambda_F=50$, 5, and 1. (a) $N=4$, (b) 7, and (c) 10. The upward arrows show the localization length.

shall calculate the spin-dependent transmission coefficients as a function of the wire length.[23,24] Using these coefficients, we calculate the spin correlation function $F_{yy}(L) \propto \langle \sigma_y(L)\sigma_y(0)\rangle$ for spin in the y direction between $x=0$ and $x=L$. The spin-relaxation length Λ_S is given by the decay rate of $F_{yy}(L) \propto \exp(-L/\Lambda_S)$. In the following we use Λ_S obtained previously.[21]

Conductance $G(L)$ of the wire can be calculated using the multichannel version[25–29] of Landauer's formula.[30] In terms of its geometric average, localization length ξ is defined as

$$\exp\langle \log G(L)\rangle \propto e^{-L/\xi}, \tag{2}$$

for sufficiently large L, where $\langle \cdots \rangle$ denotes sample average. The conductance fluctuation is defined as

$$\Delta G(L) \equiv \langle [G(L) - \langle G(L)\rangle]^2\rangle^{1/2}. \tag{3}$$

In this study, the sample average is performed over more than 2000 different impurity configurations for the purpose of suppressing statistical fluctuations. We shall choose $\lambda_F/a=7$ and vary the strength of the spin-orbit interaction as $\delta=0$ (perfect orthogonal), 0.02, 0.03, 0.05, and 0.5. As will be discussed below, a fully symplectic case is realized for $\delta=0.5$. Systematic calculations are performed for $1 \le \Lambda/\lambda_F \le 50$ and $1 \le W/\lambda_F \le 5.5$. In an InGaAs/AlGaAs quantum well characterized by electron concentration $n_s=2.0\times 10^{12}$ cm^{-2} and effective mass $m^*=0.05m_0$ with free-electron mass m_0, we have $E_F=95.8$ meV, $\lambda_F=17.7$ nm, $1.8 \le W \le 9.7$ nm, mobility $7.6\times 10^3 \le \mu \le 3.8\times 10^5$ cm^2/V s, and $\alpha=2.7\times\delta\times 10^{-10}$ eV m.

III. NUMERICAL RESULTS

Figure 1 shows examples of calculated geometric average of the conductance as a function of the length in wires with width (a) $W/\lambda_F=2.14$, (b) 3.57, and (c) 5. The upward arrows show localization length ξ. The results for $\delta=0$ and $\delta=0.5$ are qualitatively in good agreement with those obtained previously in the full symplectic quantum wires.[6,7] In narrow wires with $W/\lambda_F=2.14$ shown in (a), the conductance for $\delta=0$, 0.02, and 0.05 is nearly indistinguishable for $\Lambda/\lambda_F=1$ and a slight difference appears with the increase in Λ. In this case, the symmetry of the system remains orthogonal for both $\delta=0.02$ and 0.05 while the localization effect is reduced considerably due to full antilocalization effect in the symplectic symmetry for $\delta=0.5$.

In wider wires with $W/\lambda_F=3.57$ shown in Fig. 1(b), the conductance for $\delta=0.02$ is nearly the same as that for $\delta=0$ and a slight deviation appears for $\delta=0.05$ in dirty case $\Lambda/\lambda_F=1$. With the increase in the mean-free path, the conductance for $\delta=0.05$ approaches that for $\delta=0.5$, suggesting that the effective universality class has now changed from orthogonal to symplectic. This behavior appears also for $\delta=0.02$ in further wider wires with $W/\lambda_F=5$ shown in Fig. 1(c). Therefore, the "effective universality class" of the system depends on the mean-free path or the strength of disorder. In the following, this dependence will be shown to be determined by the relative magnitude of spin-relaxation length and the localization length.

Although not explicitly shown here, the resulting localization length for $\delta=0$ and $\delta=0.5$ is qualitatively understood in terms of the analytic but approximate expression of the localization length in a wire obtained by a Fokker-Planck equation for transmission coefficients, given by[4,5]

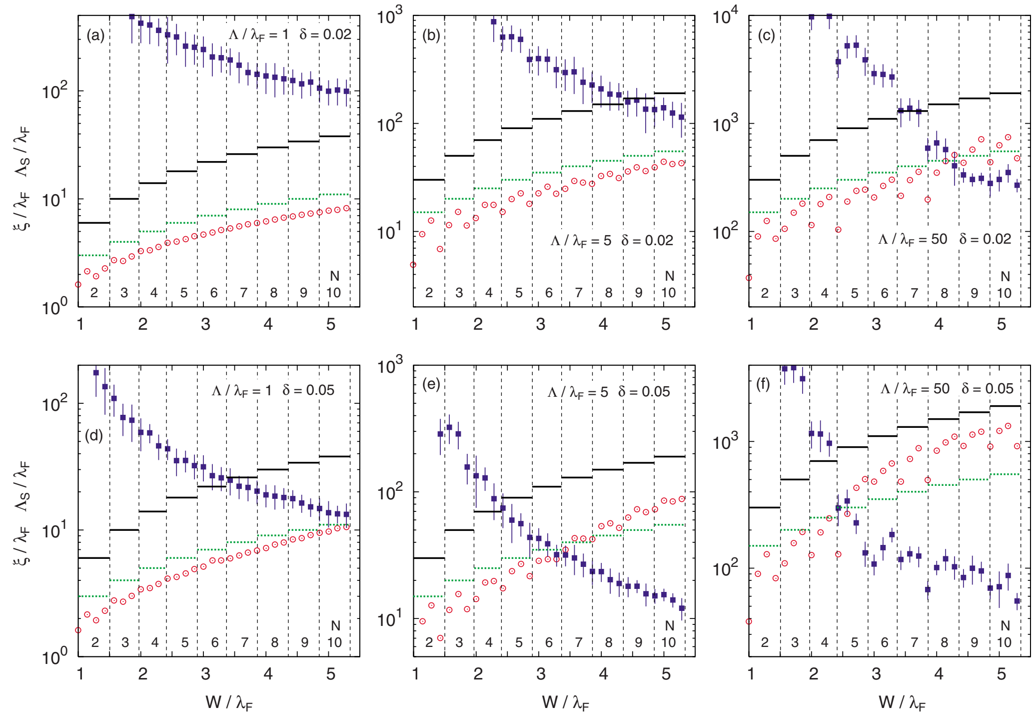

FIG. 2. (Color online) Calculated localization length ξ and spin-relaxation length Λ_S as a function of wire width W. (a) $\Lambda/\lambda_F=1$, (b) 5, and (c) 50 for δ=0.02. (d) $\Lambda/\lambda_F=1$, (e) 5, and (f) 50 for δ=0.05. The dotted lines and solid lines show Eq. (4) in the orthogonal and symplectic case, respectively. The localization length increases with W while the spin-relaxation length decreases. As a result, their relative order changes at a certain value of W depending on mean-free path Λ and δ. When Λ_S becomes smaller than ξ, the localization length starts to become larger than that of the orthogonal case and approaches that of the symplectic case.

$$\xi=\begin{cases}(N+1)\Lambda & \text{(orthogonal)},\\ (4N-2)\Lambda & \text{(symplectic)},\end{cases} \tag{4}$$

where N is the channel number. The values of localization lengths for δ=0 and 0.5 are close to the orthogonal and symplectic values in Eq. (4), respectively, while the analytic expression seems to slightly overestimate. The localization length hardly changes when δ is increased beyond 0.5, showing that the fully symplectic case is realized for δ=0.5.

Figure 2 shows calculated localization length together with spin-relaxation length as a function of the wire width for δ=0.02 and 0.05 and for Λ/λ_F=1, 5, and 50. The localization length is indicated by the open symbols and the spin-relaxation length is indicated by solid symbols. The dotted lines and solid lines show the localization length given by Eq. (4) in the orthogonal case and the symplectic case, respectively.

The localization length increases with the width while the spin-relaxation length decreases. As a result, their relative order changes at certain values of W depending on the mean-free path and δ. Once Λ_S becomes smaller than ξ, the localization length starts to become larger than that of the orthogonal case and approaches that of the symplectic case, as is particularly clear in the case δ=0.05 shown in (d)–(f). For δ=0.02, Λ_S becomes much larger and the crossover starts to occur in wider or cleaner wires as in (c) with Λ/λ_F=50. These results clearly show that the symmetry crossover occurs depending on the relative magnitude of the localization length and the spin-relaxation length rather than the strength of spin splitting.

In order to explicitly show the crossover as a function of the mean-free path, we calculate the localization length and the spin-relaxation length averaged over the wires belonging to the same channel number. The results are shown in Fig. 3. In this figure, the open symbols and the solid lines denote the localization length and the filled symbols and the dashed lines denote the spin-relaxation length, respectively. With increase in the width, the spin-relaxation length decreases, while the localization length increases, resulting in the change in the relative magnitude of Λ_S and ξ.

For N=4 shown in Fig. 3(a), the localization length increases without any features in proportion to the mean-free path. Further, it is independent of δ for δ=0–0.05 because of the "effective" orthogonal universality class and the symmetry is symplectic only for δ=0.5. For N=7 shown in Fig. 3(b), the localization length for δ=0.05 exceeds the spin-relaxation length at a particular point lying in the region $3<\Lambda/\lambda_F<4$. When the mean-free path approaches this crossing point from below, the localization length starts to deviate from that for δ=0 (orthogonal) and increases toward that for δ=0.5 (symplectic). The similar behavior can also be observed for δ=0.03. For N=10 shown in Fig. 3(c), this behavior is apparent for all of δ=0.05, 0.03, and 0.02. We can safely conclude, again, that the quantum wire with the spin-orbit interaction remains effectively as orthogonal when $\Lambda_S>\xi$ and crossovers to symplectic when $\Lambda_S<\xi$.

Figure 4 shows examples of calculated conductance fluctuation as a function of the length for various values of

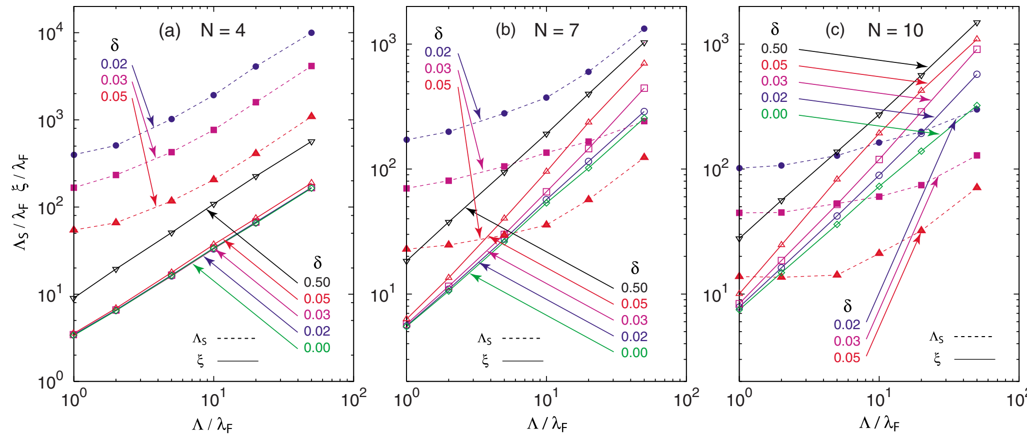

FIG. 3. (Color online) The averaged spin-relaxation length and localization length as a function of the mean-free path for channel number (a) N=4, (b) 7, and (c) 10. When $\xi \gtrsim \Lambda_S$, the localization length starts to increase faster with Λ due to the symmetry crossover from orthogonal to symplectic class.

Λ/λ_F. The localization lengths are indicated by vertical arrows and the horizontal dashed lines represent the results obtained by perturbation calculations,[31]

$$\Delta G = \frac{e^2}{h} \times \begin{cases} 0.730 & \text{(orthogonal)}, \\ 0.365 & \text{(symplectic)}. \end{cases} \tag{5}$$

We first note that the results for δ=0 (orthogonal) and 0.5 (symplectic) are essentially the same as those obtained previously.[7]

In narrow wires with channel number N=4 shown in Fig. 4(a), the results of $\delta \leq 0.05$ are all nearly the same and quite different from that of δ=0.5, showing that the system symmetry is essentially orthogonal for $\delta \leq 0.05$. This is consistent with the fact that the localization length denoted by upward arrows remains the same for $\delta \leq 0.05$ and that the spin-relaxation length is larger than the localization length independent of Λ shown in Fig. 3(a).

In wider wires with channel number N=7 shown in Fig. 4(b), the curves for δ=0.05 become closer to those for δ =0.5 with the increase in Λ, suggesting that the symmetry

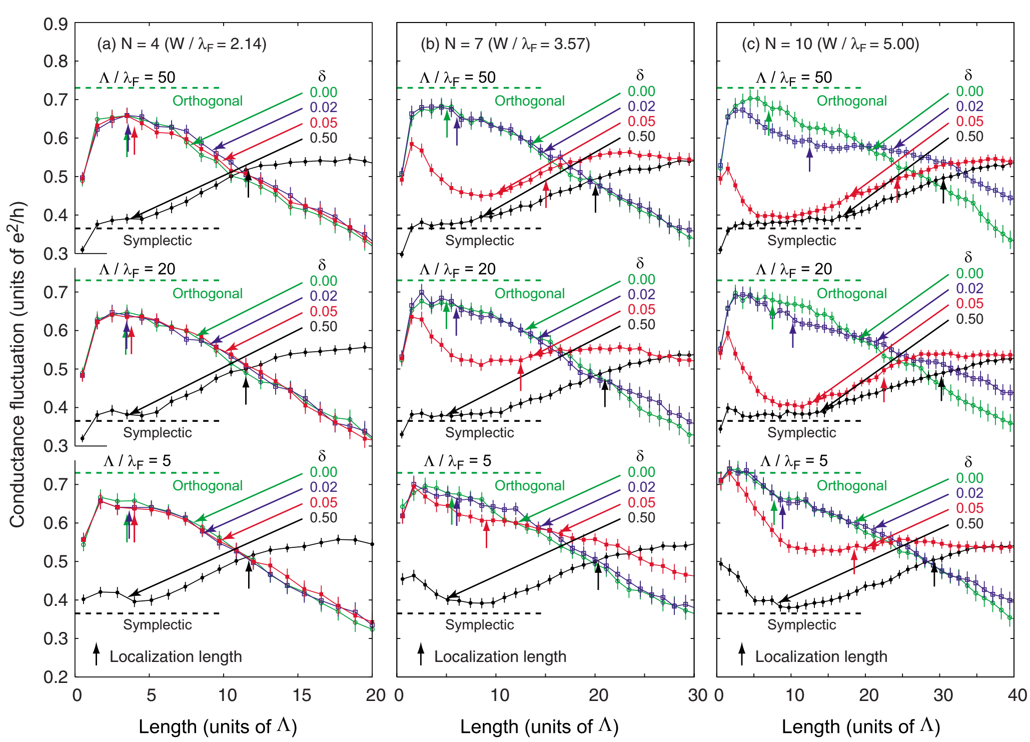

FIG. 4. (Color online) Calculated conductance fluctuation of wires as a function of the length for Λ/λ_F=50, 20, and 5. (a) N=4, (b) 7, and (c) 10. The horizontal dashed lines represent the perturbation results and the vertical arrows denote the localization length. When the localization length deviates from that of orthogonal case, the conductance fluctuation starts to change its dependence on the length, approaching the result for fully symplectic δ=0.5.

crossover is taking place there. This is again consistent with the behavior of the localization length, which shifts from the value of $\delta=0$ to that of $\delta=0.5$ in a similar manner. In even wider wires with $N=10$ in Fig. 4(c), the symmetry crossover for $\delta=0.05$ occurs in the region of smaller Λ, and the symmetry change starts to appear even for $\delta=0.02$ with the increase in Λ. This behavior of the conductance fluctuation again shows that the spin-relaxation length is the relevant length changing the effective symmetry class from orthogonal to symplectic.

The role of the spin-relaxation length in the symmetry crossover can be understood as follows: In the quantum wire, electron wave function can extend only within a finite region determined by the localization length. If the spin-relaxation length is much larger than the localization length, an electron does not forget the initial spin direction before the wave function completely decays. In such a case, effects of the spin-orbit interaction cannot have any influence on the states and therefore the system remains in the effective orthogonal class. When the localization length exceeds the spin-relaxation length, however, the spin memory is destroyed within the extent of the localized wave function and the symmetry crossover fully develops from orthogonal to symplectic.

IV. SUMMARY

In this paper, we calculated the localization length and the conductance fluctuation in quantum wires with spin-orbit interaction, and showed that the effective universality class is determined by the relative magnitude of the spin-relaxation length and the localization length, irrespective of individual parameters such as wire width, strength of spin-orbit interaction, or mean-free path. When the spin-relaxation length is longer than the localization length, the universality class of this system is orthogonal. When the spin-relaxation length becomes comparable to or smaller than the localization length, the symmetry crossovers to symplectic class.

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